User Manual for 2dTiler V2.0

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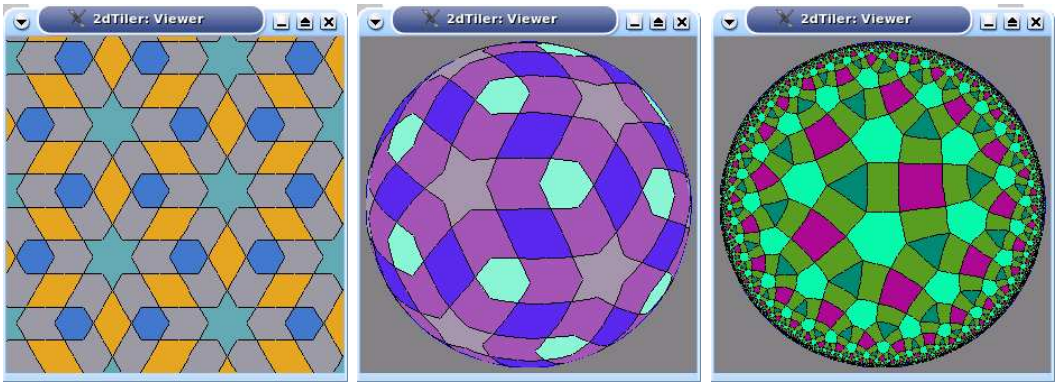

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1 Overview

2dTiler is an interactive viewer for periodic tilings of all three two-dimensional geometries, written by Daniel Huson and Klaus Westphal. Input is a file of records, each describing a Delaney-Dress symbol, a data-structure that prescribes the topology and symmetries of a periodic tiling, and output is the picture of that tiling. The geometric shape of the tiles and the specific symmetry group can be interactively modified.

2 Installation

To install the program under Linux, download the archive file `http://www-ab.informatik.uni-tuebingen.de/software/2dtiler/2dtiler20.zip` and unpack it using the command `unzip 2dtiler20.zip`.

this will produce a directory `2dtiler` containing the following:

- a file `manual.pdf` containing a user manual for the program,
- a script `2dt` to start the program from outside of the `2dtiler` directory,
- the executable program `2dt.exe`,
- a directory `lib` containing a compiled version of `TCL-TK` (8.0.5), and
- a directory `tilings` containing a number of input files for the program.

To start the program, `cd 2dtiler` and then type `2dt.exe`. If you would like to be able to start the program from outside of the `2dtiler` directory, then please set an enviroment variable `TILERDIR`

to point at the directory, e.g. `setenv TILERDIR ~/2dtiler`, and then copy the file `2dtiler/2dt` to your bin directory. Then you can use `2dt` to start the program.

Important: The program requires TCL-TK version 8.0.5. (Precisely this version, not an earlier or later one!). A compiled version of this is provided with the program, and should work as long as you start the program as described above. If this does not work, please download TCL (8.0.5.) from `ftp://www.scriptics.com/pub/tcl/tcl8_0/tcl8.0.5.tar.gz` and TK (8.0.5) from `ftp://www.scriptics.com/pub/tcl/tcl8_0/tk8.0.5.tar.gz`, and install both packages on your computer.

3 Getting started

Type `./2dt` to start the program. This opens two windows, the `Viewer` and `Controls` windows. Use the `File→Open Collection` item of the `Controls` window and select a file from the `tilings directory`. The first tiling (of many) will appear in the `Viewer` window. Use the `Tilings` menu to navigate through the file. (Alternatively, you can press the space-bar or 'b'-key to move forward or backward in the file.) Use the items in the `Edit` and `Layout` menus to modify the appearance of tilings. Alternatively, to reshape tilings, press the 'a'-key while in the `Viewer` window. This will select and highlight all movable vertices and movable edge centers. Click and drag these to reshape. Use the `Modify→Mutate` item to modify the symmetry group of a tiling. Modified tilings can be saved in their current state using the `File→Save` item.

4 Mathematical background

The program is based on "combinatorial tiling theory". In this approach, due to Andreas Dress, each tiling is encoded in terms of its *Delaney-Dress symbol*, which is essentially a finite, connected, edge-colored and vertex-labeled graph that precisely captures the topology and symmetries of a *tiling*. The input files of the program consist of systematically enumerated symbols. The program reads a symbol from the input file, constructs a *fundamental domain* for the tiling, computes *generators* for the *symmetry group* and then applies these to the fundamental domain to obtain the whole tiling. See the references for more details.

5 Windows

The program has two main windows, the `Viewer` window and the `Controls` window. All drawing takes place in the `Viewer` window, whereas all user menus are contained in the Controls window.

(The reason why the menus are not attached to the `Viewer` window is that the `Window` is imple-

mented using OpenGL, whereas the user menus are written in tcl-tk and when the program was written it was not possible to attach a tcl-tk menu bar to an OpenGL window).

By selecting the Modify→Mutate menu item, the user can open an additional Mutate window. For a symmetry group containing *rotations* or *dihedral symmetries*, this windows allows modification of the rotational orders and thus of the symmetry group of the displayed tiling.

All menus are tear-off menus and this can also be considered as individual windows.

6 Files

The program works with four types of files:

Files ending on .tgs contain a *collection* of symbols, one for each tiling, and no additional information. These files are opened using the File→Open Collection menu item.

Files ending on .2dt each contain precisely one *modified tiling*, that is, a tiling and additional information including all modifications interactively made by the user. The files are opened using the File→Open menu item.

Files ending on .2tc contain a color palette which is used to color tilings. These files are opened and saved using the Tile Colors→Open and Tile Colors→Save in the Tile Colors submenu of the Edit menu.

Additionally, the menu item File→Create PS produces a .ps file containing a *postscript* picture of the current tiling.

7 Menus

Note that all menus can be torn-off (opened as a separate window) by selecting the dotted line at the top of the menu. In the following we discuss each menu and the menu items it contains.

7.1 File menu

The File menu of the Controls window contains the following menu items:

- The File→New Window item starts a second copy of the program, but only works if the program is called 2dt and is in your path.
- The File→Open item opens a modified tiling, file ends on .2dt.
- The File→Open Collection item opens a collection of tilings, file ends on .tgs.

- The `File→Save` item saves the current tiling in its current state as a modified tiling to a file ending on `.2dt`.
- The `File→Export PS` item produces a postscript picture of the current tiling and exports it to a file.
- The `File→To Geomview` items saves the current tiling in Geomview format.
- The `File→Commands` item open the command-line interface to the program. Unfortunately, the commands are currently undocumented. One is of particular interest: `dim 2` or `dim 3` change the dimension of the space in which the tiling is draw. For example, use `dim 2` to see the stereographic projection of a spherical tiling or use `dim 3` to see the hyperboloid picture of a hyperbolic tiling.
- The `File→Quit` item terminates the program.

7.2 Edit menu

The `Edit` menu contains the following commands:

- The `Edit→Undo` item is not implemented.
- The `Edit→Cut` item is not implemented.
- The `Edit→Copy to XFig` item copies the current tiling to the paste buffer of the `xFig` program.. For this to work, you must have `pstoedit` installed (freely obtainable on the web).
- The `Edit→Paste` item is not implemented
- The `Edit→Line Select` item contains a submenu that allows you to `select all`, `select all visible` or `select all invisible` lines, or to `deselect all` lines. A line is invisible if you have given it width `none`. A selected edge is highlighted in red.
- The `Edit→Line Width` item is used to set the line width of all currently selected edges to the selected number, or to `none`.
- The `Edit→Straighten` item pulls all selected, or all edges, straight. If `Always` is on, then the program will alway straighten a tiling when first drawing it.
- The `Edit→Tile Colors` submenu contains a number of different items. The `Tile Colors→Show` item toggles between displaying colors or not. The `Tile Colors→Color` item sets to color mode, the `Tile Colors→Black & White` item sets to black and white mode, and the `Tile Colors→Random` item selects new random colors. The `Tile Colors→Open` and `Tile Colors→Save` items are used to open or save a color palette (a `.2tc` file).

7.3 Tilings menu

The `Tilings` menu is used to move through a collection of tilings:

- The `Tilings→Next` item moves to next tiling in file.
- The `Tilings→First` item moves to first tiling in file.
- The `Tilings→Previous` item moves to previous tiling in file.
- The `Tilings→Last` item moves to last tiling in file.
- The `Tilings→Search` item is not implemented.

7.4 Layout menu

The `Layout` menu controls layout-related aspects of the program:

- The `Layout→Show F. Domain` items highlight the fundamental domain.
- The `Layout→Show Chambers` item shows the *chamber system* .

The next three items determine the *drag mode* , which determines the effect of dragging the mouse in the `Viewer` window:

- The `Layout→Zoom mode` item switches to *zoom mode* .
- The `Layout→Rotate mode` item switches to *rotate mode* .
- The `Layout→Translate` item switches to *translate mode* .

The next two items effect only hyperbolic tilings:

- The `Layout→Show More Tiles` item draws twice as many tiles.
- The `Layout→Show Less Tiles` item draws half as many tiles.
- The `Layout→Reset Layout` item reverts to the default layout.

7.5 Modify menu

The `Modify` menu contains items that can be used to alter the current tiling (and the corresponding Delaney-Dress symbol):

- The `Modify→Mutate` item opens the `Mutate` window.
- The `Modify→Dualize` item is used to switch to the *dual tiling* .
- The `Modify→Max Symmetry` item shows the tiling with *maximal symmetry* .
- The `Modify→Orientate` item can be used to remove all *orientation reversing symmetries* from the symmetry group, if present.

The next three items determine the *modification mode* effect of the program, that is, what happens when the mouse is clicked on a selected edge or selected vertex:

- The `Modify→Reshape mode` item selects the *reshape mode* in which vertices and edge centers can be dragged and repositioned.
- The `Modify→Truncate Vertex Mode` item selects the *truncate vertex mode* in which clicking a selected vertex will truncate that vertex.
- The `Modify→Contract Edge Mode` item selects the *contract edge mode* in which clicking a selected edge will contract that edge (if possible).

7.6 Help menu

The `Help` menu only contains one item:

- The `Help→About` item reports the version number.

8 Mouse interaction

Dragging the mouse in the `Viewer` window will either zoom, rotate or translate the current tiling, depending on the current mode, see

Clicking on an edge or vertex will select it, if it was unselected and is selectable. If it is selected, it will either allow reshaping by dragging, it will truncate a vertex or it will contract an edge, depending on the modification mode.

9 Keyboard shortcuts

In the `Controller` window, the menus can be navigated using `Alt-key` short cuts, using the underlined letters in the menus. The `Viewer` window recognizes the following keyboard short cuts:

- `spacebar-key`: move to next tiling
- `'a'-key`: select all edges
- `'b'-key`: move to previous tiling
- `'B'-key`: show/hide chamber system (barycentric subdivision)
- `'c'-key`: show/hide colors
- `'C'-key`: change modification mode to contract edge mode
- `'d'-key`: randomly change colors
- `'D'-key`: dualize
- `'f'-key`: show/hide fundamental domain
- `'G'-key`: save tiling in `Geomview` format
- `'h'-key`: color in black and white ("heaven and hell mode")
- `'I'-key`: command-line interface: read command from `stdin`
- `'l'-key`: draw less tiles (hyperbolic case)
- `'m'-key`: draw more tiles (hyperbolic case)
- `'o'-key`: reset layout
- `'O'-key`: orientate tiling (if possible)
- `'Q'-key`: quit
- `'r'-key`: change drag mode to rotate mode
- `'R'-key`: change modification mode to reshape mode
- `'s'-key`: straighten all edges
- `'t'-key`: change drag mode to translate mode
- `'T'-key`: change modification mode to truncate vertex Mode
- `'x'-key`: compute tiling with maximal symmetry
- `'z'-key`: change drag mode to zoom mode
- `'['-key`: move to first tiling in file
- `']'-key`: move to last tiling in file
- `'<'-key`: move to previous tiling in file
- `'>'-key`: move to next tiling in file
- `'0'-key` to `'9'-key`: change width of selected edges to this number

10 Bugs and unusual features

- Before showing a tiling, the program computes an internal display list of all objects. If you have zoomed a Euclidean tiling to a very small size or have requested that the program shows very many hyperbolic ones, then it might take minutes before the picture appears. Unfortunately, there is currently no mechanism in place to interrupt this without killing the program.

- The program can not display any spherical tiling with trivial symmetry group.

- Rendering of spherical tilings is very poor. For nice pictures of them, use the `Geomview`.

- Many others

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